Q.P. Code : 11323

Third Semester B.Sc. Degree Examination, November/December 2019

(Semester Scheme – CBCS – Freshers – 2019-20)

MATHEMATICS – III

Time : 3 Hours] *[Max. Marks : 70*

Instructions to Candidates : Answer all questions.

PART – A

Answer any **FIVE** questions : **(5 × 2 = 10)**

1. (a) Define order of an element of a group.

   (b) Find the number of generators of a cyclic group of order 24.

   (c) Test the nature of the sequence $\{1+\cos n\pi\}$.

   (d) State Raabe's Test for series of Positive terms.

   (e) Test the convergence of the series

   $$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\dots$$

   (f) State Rolle's theorem.

   (g) Find Left hand Limit of

   $$f(x)=\begin{cases}1+x & \text{for } x\le 2\\ 5-x & \text{for } x>2\end{cases} \text{ at } x=2.$$

   (h) Evaluate $\lim_{x\to 0}\left(\frac{a^x-b^x}{x}\right)$.

PART – B

Answer any **ONE** full question : **(1 × 15 = 15)**

2. (a) Prove that if 'a' is any element of a group G of order n then $a^m=e$ for any integer m if and only if n divides m.

   (b) Prove that subgroup of a cyclic group is cyclic.

   (c) Find all the right cosets of the subgroup $H=\{0, 4, 8\}$ in Z_{12} and also verify Lagrange's theorem.

Or

3. (a) If a and x are any two elements of a group G then prove that $O(a) = O(x a x^{-1})$.

(b) Prove that the group $G = \{1, 2, 3, 4, 5, 6\}$ under multiplication modulo 7 is cyclic and find its generators.

(c) If G is any finite group and H is any subgroup of G then prove that $O(H)$ divides $O(G)$.

PART – C

Answer any **TWO** full questions : **(2 × 15 = 30)**

4. (a) Prove that a monotonically increasing sequence which is bounded above is convergent.

(b) Discuss the nature of the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$.

(c) Examine the convergence of the sequences :

(i) $\left\{\sqrt{n+1}-\sqrt{n}\right\}$

(ii) $\left\{\frac{(n+1)^{n+1}}{n^n}\right\}$.

Or

5. (a) Prove that every convergent sequence is bounded.

(b) If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$ then prove that $\lim_{n\to\infty}(a_n + b_n) = (a+b)$.

(c) Discuss the nature of the sequence $\{a_n\}$ where $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$.

6. (a) State and prove D'Alembert's Ratio Test for series of positive terms.

(b) Test the Convergence of the series $1 + \frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \dots \infty$.

(c) Sum the series to infinity

$$1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \dots$$

Or

7. (a) State and prove Cauchy's Root Test for series of positive terms.

(b) Discuss the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{nx}{n+1}\right)^n$.

(c) Sum the series to infinity $\frac{1^2}{1!} + \frac{2^2}{2!} + \frac{3^2}{3!} + \dots$

PART – D

Answer **ONE** full question : **(1 × 15 = 15)**

8. (a) Prove that a function which is continuous in a closed interval attains its bounds.

   (b) Verify Rolle's theorem for the function $f(x) = x(x-3)^2$ in $[0, 3]$.

   (c) Evaluate $\lim_{x \to \frac{\pi}{2}} (\sec x)^{\cot x}$.

Or

9. (a) Examine the differentiability of the function $f(x) = \begin{cases} x^2 & \text{if } x \le 3 \\ 6x-9 & \text{if } x > 3 \end{cases}$ at $x = 3$.

   (b) State and prove Lagrange's mean value theorem.

   (c) Using Maclaurin's expansion prove that

   $$\log(1+x) = 1 - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$